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Criteria for the Onset of Significant Concentrations of Ice Particles in Cumulus Clouds

ARTHUR L. RANGNO and PETER V. HOBBS

Department of Atmospheric Sciences, University of Washington, Seattle, WA 98195 (U.S.A.)

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ABSTRACT

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Field observations and measurements, reported from many locations around the world, are used to deduce the cloud depths (and cloud top temperatures) required for the Onset of Significant Concentrations of Ice Particles (OSCIP) in cumulus clouds. The results show that the OSCIP will generally occur if cloud top temperatures are between about −4° and −10°C. The results are compared with the criteria for the OSCIP proposed by Hobbs and Rangno (1985) and for the formation of high ice particle concentrations by riming-splintering.

RESUME

A partir d'observations et de mesures sur le terrain en de nombreuses régions du monde, on évalue les épaisseurs de nuages et les températures à leur sommet nécessaires à l'apparition de concentrations importantes en particules de glace dans les cumulus. Les résultats montrent que de telles concentrations apparaissent habituellement pour des températures au sommet du nuage comprises entre −4°C et −10°C. On compare ces résultats aux critères proposés par Hobbs et Rangno (1985), et à la formation de fortes concentrations en particles de glace par givrage et génération d'éclats.

INTRODUCTION

The formation and growth of ice in clouds is one of the important mechanisms by which precipitation is produced. It is, therefore, of considerable importance to establish the conditions under which the Onset of Significant Concentrations of Ice Particles (OSCIP) occurs in clouds. By significant concentrations we mean on the order of 1 per liter or greater.

Ice nucleus measurements indicate that the formation of significant concentrations of ice particles by so-called *primary processes* of ice formation requires temperatures at or below about −20°C (see, for example, Fletcher, 1962).

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However, in many clouds the concentrations of ice particles that ultimately form greatly exceed 1 l^{-1} at temperatures well above −20°C. Such high concentrations of ice particles are said to form by *secondary processes* (or *ice enhancement*), although some form of nucleation might still be involved (e.g., Young, 1974; Mossop, 1985; Hobbs and Rangno, 1985; Rosinski et al., 1986).

In the present paper we use field observations and measurements, reported by many workers from many locations around the world, to deduce the cloud depth or cloud top temperature (T_T) required for the OSCIP as a function of cloud base temperature (T_B) for cumulus clouds. We then compare these deductions with some of the criteria that have been proposed for OSCIP.

DEDUCTIONS FROM FIELD OBSERVATIONS

The observations we have used are listed in Table I. In most cases definitive information on the conditions for the OSCIP is not given in the original papers, but it can generally be inferred. For example, in an early report by Murgatroyd and Garrod (1960) no ice was found in a cumulus cloud with a top temperature of −8°C. However, on the same day, they reported ice particles in concentrations of about 100 l^{-1} in a cumulus cloud with a top temperature of −11°C. On another day, when they sampled more than one cloud, ice was found in a cloud with a top temperature of only −6°C. We therefore estimate from their data that the OSCIP was about −9°C. In the case of the data of Morris and Braham (1968), we have taken as the temperature for OSCIP the cloud top temperature (−8°C) where 50% of the dissipating clouds contained ice particles. (We did this since 100% of the dissipating clouds with top temperatures between −11° and −12°C contained ice, no doubt with average concentrations far exceeding 1 l^{-1}. Thus, the OSCIP must have occurred at a substantially warmer cloud top temperatures.)

Due to the fact that young, rising turrets in cumulus clouds are often ice-free but may develop considerable ice some time later (e.g., Koenig, 1963), there is a tendency to underestimate the ice-forming capabilities of cumulus clouds. Therefore, if it were reported, for example, that one-third of the clouds sampled at a particular temperature contained ice but no information was given on the stages in the lifecycles (growing, mature and dissipating) of the clouds when the measurements were made, or if the sampling was largely within younger clouds, we would associate this temperature, and the corresponding cloud depth, with the OSCIP. This was done for the data of Schemenauer and Isaac (1984). Also, if ice is reported as characteristic of a particular cloud top temperature in an airborne study but sensitive instrumentation was not used for its detection (e.g., Coons and Gunn, 1951), or if ice was deduced from careful visual observations (e.g., Ludlam, 1955), we have assumed that ice was present in concentrations of at least 1 l^{-1}, that is, the OSCIP had occurred.

Observations made prior to the development of sensitive instrumentation for the detection of ice in clouds are listed in Table I if they are supported by more recent measurements in similar clouds. For example, the report from Coons et al. (1949) of ice forming in cumulus clouds with top temperatures of $-7\,^\circ$C along the Gulf Coast has since received support from measurements in tropical cumulus (e.g., Hallett et al., 1978). Similarly, Ludlam's visual assessment of ice in small polar maritime cumuli with tops at $-5\,^\circ$C is supported by the measurements of Mossop et al. (1967, 1968, 1970, 1972) and Hobbs and Rangno (1985).

In general, the cloud top temperatures for the OSCIP given in Table I are probably accurate to $\pm 2^\circ$.

We have divided the clouds into continental and maritime, based on consideration of their location, airflow and droplet concentrations (when available). This may lead to some differences from previous classifications. For example, we consider the cumulus clouds sampled by Koenig (1963) over Missouri to be continental, although Brown and Braham (1963) termed them maritime.

Also shown in Table I is the airmass type (polar or tropical) in which the clouds formed. We define polar air as having an equivalent potential temperature, $\theta_E \leqslant 315\,^\circ$C and tropical air as having $\theta_E \geqslant 325\,^\circ$C.

Shown in Fig. 1 are the cloud depths required for the OSCIP plotted against cloud base temperature for the continental cumulus clouds listed in Table I(a). Apart from two points (numbered 8 and 10), which will be discussed later, the other eighteen points show an excellent correlation ($r=0.95$) between cloud base temperature and cloud depth for the OSCIP. The data for maritime cumulus clouds (Table I(b)) are plotted in Fig. 2. Although the number of data points is small, these points indicate that the OSCIP occurs at rather uniform temperatures in maritime air masses when cloud base temperatures are $>0\,^\circ$C.

The results for continental cumulus clouds are shown in a different form in Fig. 3, where the cloud top temperature required for the OSCIP is plotted against cloud base temperature. This plot indicates that for cloud base temperatures $\gtrsim 10\,^\circ$C the OSCIP occurs at cloud top temperatures of about $-7\,^\circ$C, but as the cloud base temperature decreases below about $10\,^\circ$C the cloud top temperature required for the OSCIP slowly decreases.

It can be seen from Figs. 1 and 3 that the data points labelled 8 and 10 are "outliers". These data points indicate significantly lower (i.e. colder) cloud top temperatures for the OSCIP than the rest of the data points.

Data point 8 is that reported by Gagin (1975) for continental cumulus clouds in Israel. Gagin reported that with cloud base temperatures of $\sim 8\,^\circ$C these clouds did not exhibit the OSCIP until the cloud top temperatures fell, on average, to $-17\,^\circ$C. However, there are several reasons why Gagin might have underestimated the concentrations of ice particles in the clouds he studied. Firstly, he reports that "preference was given to clouds having smooth, 'hard' isolated tops", and that sampling was terminated when the "cloud tops showed signs of decay". As Koenig (1963) and Hobbs and Rangno (1985) have noted,

TABLE I

Summary of deductions from field observations of conditions for the onset of significant concentrations of ice particles (OSCIP) in cumulus clouds

Code number*[1]	Location	Airmass type	Characteristic cloud base temperature (°C)	Characteristic cloud top temperature for OSCIP (°C)	Characteristic cloud depth for OSCIP (km)	Number of clouds sampled	Reference
(a) *Continental cumulus*							
1	Southern Alabama	Tropical	20	−7	5.2	39	Coons et al. (1949); Coons and Gunn (1951)
2	Ohio	Tropical	15	−8	4.3	46	Coons et al. (1949); Coons and Gunn (1951)
3	Massachusetts	Tropical	11	−7	3.1	8	Plank et al. (1955)
4	Southern Missouri	Tropical	15	−7	4.2	84	Koenig (1963)
5	Minnesota	Tropical	11	−8	3.3	92	Morris and Braham (1968)
6	Northern Arizona	Tropical	5	−10	2.6	7	MacCready and Takeuchi (1968); Takeuchi (1970)
7	Southeast Australia	Polar	8	−6	2.6	23	Mossop et al. (1972)
8	Israel	Polar	8	−17	3.9	74	Gagin (1975)
9	Southern Florida	Tropical	21	−7	5.3	4	Hallett et al. (1978)
10	Northeast Colorado	Tropical	1	−22	3.3	12	Heymsfield et al. (1979)
11	Northeast Colorado	Tropical	0	−14	2.2	1*[2]	Paluch and Breed (1984)
12	Northwest Territories	Polar	3	−12	2.4	58	Isaac and Schemenauer (1979)
13	Ontario, Canada	Tropical	11	−11	3.9	54	Schemenauer and Isaac (1984)
14	Eastern Montana	Tropical	4	−9	2.4	93	Hobbs et al. (1978); Hobbs et al. (1980)

15	Eastern Montana	Tropical	4	−12	2.6	32	Schemenauer and Isaac (1984)
16	Eastern Montana	Tropical	4	−11	2.5	8	Cooper and Lawson (1984)
17	Oklahoma	Tropical	16	−7	4.2	18	Heymsfield and Hjelmfelt (1984)
18	Cascade Mountains (Washington)	Tropical	5	−10	2.4	169	Hobbs and Rangno (1985)
(b) *Maritime cumulus*							
19	Western Sweden	Polar	2	−5	1.3	"Several"	Ludlam (1955)
20	Southern England	Polar	2	−9	1.8	"Many" but exact number not given	Murgatroyd and Garrod (1960)
21	Pacific Ocean off Southern Australia	Polar	4	−6	1.6	> 100	Mossop (1978a, 1985); Mossop et al. (1967, 1968, 1970, 1972); Mossop and Ono (1969)
22	Island of Hawaii	Tropical	~18	−5	4.6	"Several"	M. Baker (private communication, 1985)
23	Washington State Coastal Waters	Polar	3	−6	1.5	347	Hobbs and Rangno (1985)
24	Puget Sound and Western Washington State	Polar	0	− 8	1.4	408	Hobbs and Rangno (1985)
25	Cascade Mountains and Eastern Washington State*[3]	Polar	−10	−17	1.1	371	Hobbs and Rangno (1985)

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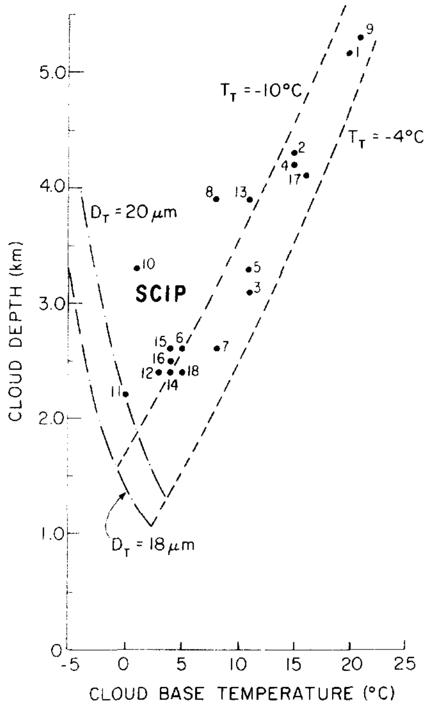
*[1] This is the number used to indicate the datum point in Figs. 1–4.

*[2] The authors state that this case study was representative of several other days.

*[3] In polar westerly airstreams the cloud droplet size distributions in Eastern Washington reveal maritime characteristics, even though the total concentration of droplets may exceed normal maritime values.



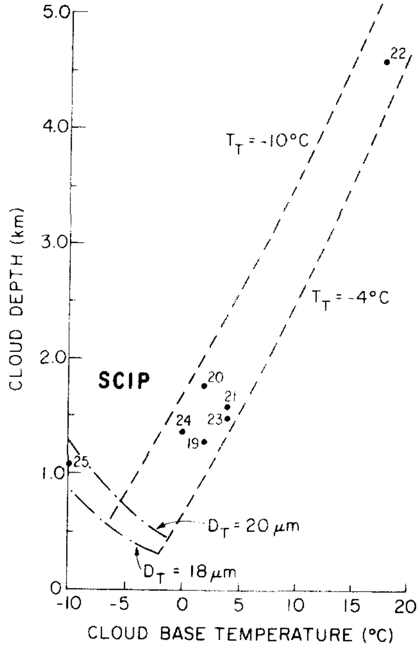


Fig. 1. Field observations for the onset of significant concentrations of ice particles (OSCIP) in continental cumulus clouds (the points are labelled by code numbers; see Table I for information on each code number). The dashed lines show the −4° and −10°C isotherms computed assuming saturated adiabatic conditions. The dashed-dotted lines are $D_T = 18\,\mu m$ and $D_T = 20\,\mu m$, computed as described in the text. The Hobbs-Rangno criteria predict that the conditions for OSCIP lie on or between the $D_T = 18\,\mu m$ and $D_T = 20\,\mu m$ lines and on or between the $T_T = -4°C$ and $T_T = -10°C$ lines. Thus, significant concentrations of ice particles (SCIP) are predicted to occur in the wedge-shaped regions between these lines.

Fig. 2. As for Fig. 1 but for maritime cumulus clouds. See Table I for information on each code number.

ice tends *not* to form in young rising cumulus turrets (which are characterized by smooth, hard tops); instead, ice appears in the later stages in the lifecycle of a turret as it begins to decay and mix with ambient air at cloud top. Secondly, Gagin did not count isolated ice particle fragments, for fear that they might have been produced impaction (on a Formvar replicator) during collection. However, there is mounting evidence that ice fragments are a ubiquitous feature of clouds (e.g., Koenig, 1963; Hobbs and Farber, 1972; Hobbs and Rangno, 1985). Gagin estimated that the rejection of ice fragments could have reduced the ice particle concentrations by an order of magnitude. If the fragments are not excluded, the discrepancy between Gagin's measurements and the prepon-

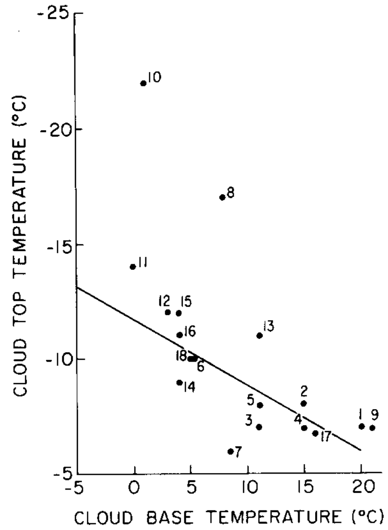


Fig. 3. Field observations of cloud top temperatures versus cloud base temperature for the OSCIP in continental cumulus clouds (the points are labelled by code numbers; see Table I for information on each code number). The line is the best-fit curve ($r = 0.77$) to the data points excluding points 8 and 10.

derance of the data points in Figs. 1 and 3 is significantly reduced. Finally, a recent study of precipitating clouds in Israel (Rangno, 1988) indicated that precipitation falls from clouds with top temperatures warmer than −10°C; these observations suggest that the OSCIP in these clouds occurs at temperatures considerably warmer than that reported by Gagin, and close to the best-fit curve shown in Fig. 3.

Data point 10 in Figs. 1 and 3 is from Heymsfield et al. (1979), who reported that continental cumulus clouds with base temperatures of ~1°C in northeast Colorado did not exhibit the OSCIP until cloud top temperatures reached about −22°C, which corresponds to a cloud depth of ~3.3 km. However, Heymsfield et al. were primarily interested in ice particle formation in the most vigorous cumulus updrafts. As noted above, ice formation appears to be suppressed, occasionally to very low temperatures, in strong updrafts. Also, in strong updrafts, ice particles may be carried to considerable heights above the level of origin before they attain detectable sizes. Paluch and Breed (1984) reported ice particle concentrations of ~1 l^{-1} at a cloud depth of 2.3 km and a temperature of −14°C (and 100 l^{-1} at −18°C) in a cloud in northeast Colorado that had a base temperature of 0°C, and they indicated that this observation was typical of a number of clouds studied by them. This measurement (point 11 in Figs. 1 and 3) is in good agreement with the majority of the other data points. Finally, we note that radar and visual observations of clouds in northeast Colorado by Knight et al. (1983) yield cloud depths and temperatures for the OSCIP that are similar to the $-14\,^{\circ}\mathrm{C}$ level measured by Paluch and Breed.

COMPARISONS OF FIELD OBSERVATIONS WITH THE HOBBS-RANGNO CRITERIA FOR ICE ENHANCEMENT

In a recent paper (Hobbs and Rangno, 1985) we presented an extensive new set of measurements of ice particle concentrations in clouds. From this data set we deduced that the OSCIP occurred near the tops of the clouds when the temperature there fell to $-7^{\circ} \pm 3\,^{\circ}\mathrm{C}$ provided that the threshold diameter (D_{T})[*1] of the droplet spectrum was $\gtrsim$ 18–20 μm. Unfortunately, droplet size information is not available for most of the observations listed in Table I. Therefore, we will recast the Hobbs-Rangno (H-R) criteria in terms of the dependence on cloud base temperature of the cloud depth required for the OSCIP. This is easily done for the first of the H-R criteria, since the heights above cloud base of the $-7^{\circ} \pm 3\,^{\circ}\mathrm{C}$ isotherms can be determined by using the saturated adiabatic lapse rates.

The other H-R criterion requires a D_{T} value of at least 18–20 μm near cloud top. Since this criterion was deduced by Hobbs and Rangno from measurements made near the tops of young cumulus turrets, prior to significant entrainment or the formation of ice in these regions, and since the growth of droplets by the collision–coalescence process is weak in such turrets, we can assume that the droplets grew by condensation alone. In this case, the diameter D of a droplet at time t is given by (e.g., Wallace and Hobbs, 1977):

$$D\frac{\mathrm{d}D}{\mathrm{d}t} = \frac{4D_{\mathrm{v}}\rho_{\mathrm{v}}}{\rho}\frac{S}{100} \tag{1}$$

where D_{v} is the diffusion coefficient of water vapor in air (taken to be 0.30 cm^{2} s^{-1}), ρ_{v} the density of water vapor in the ambient air, ρ the density of liquid water (1 g cm^{-3}), and S the percentage supersaturation of water vapor in the ambient air. In the presence of a steady updraft $u = \mathrm{d}h/\mathrm{d}t$ where h is the height above cloud base, eq. 1 becomes:

$$D\,\mathrm{d}D = \frac{D_{\mathrm{v}}\rho_{\mathrm{v}}S}{25\,\rho u}\,\mathrm{d}h \tag{2}$$

or, if the integration is carried out over sufficiently small height increments ΔH, and $\bar{S}$ is the mean supersaturation over ΔH:

$$\int_{D}^{D+\Delta D} D\,\mathrm{d}D = \frac{D_{\mathrm{v}}\rho_{\mathrm{v}}\bar{S}}{25\,\rho u}\int_{H}^{H+\Delta H} \mathrm{d}h \tag{3}$$

[*1] D_{T} is defined such that the cumulative concentrations of droplets with diameters $\geqslant D_{\mathrm{T}}$ is 1 cm^{-3}.

where D is the diameter of the droplet at height H above cloud base, and $D + \Delta D$ the diameter at height $H + \Delta H$. Hence:

$$\Delta D^2 + 2D\,\Delta D = \frac{2D_v \rho_v \bar{S}}{25\,\rho u}\,\Delta H \tag{4}$$

We can use eq. 4 to obtain D_T as a function of cloud base temperature and cloud depth by considering droplets that start out from cloud base by condensing onto cloud condensation nuclei (CCN) that are present in concentrations of $\sim 1\ \text{cm}^{-3}$. We will assume that in maritime and continental air masses particles with diameters of $\sim 4\ \mu\text{m}$ and $\sim 8\ \mu\text{m}$, respectively, are present in concentrations of $\sim 1\ \text{cm}^{-3}$. Therefore, these are taken to be the diameters of the droplets at cloud base for maritime and continental clouds, respectively. The height increment ΔH is taken to be 300 m and u is taken to be 5 m s^{-1}.

The CCN spectrum depends primarily on the type of airmass. Since precise values of $\bar{S}$ are unknown, and the computed values of D_T are sensitive to slight changes in $\bar{S}$, we "tuned" the values of $\bar{S}$ for continental and maritime clouds until they produced changes of D_T with height that were similar to our measurements for these two types of clouds. The values of $\bar{S}$ that produced the best agreement were 0.1% for continental clouds and 0.35% for maritime clouds. The curves computed from eq. 4 using these values, and for several cloud base temperatures, are shown as the dashed lines in Fig. 4, where they can be compared with measured values (the solid lines in Fig. 4).*[1]

We can now use the results shown in Fig. 4 to determine approximate cloud depths, for specified cloud base temperatures, required to achieve a D_T of 18–20 μm. These results are shown as the dashed-dotted lines in Figs. 1 and 2. Hence, based on the H-R criteria, the combinations of conditions required for the OSCIP lie on or between the dashed lines and on or between the dashed-dotted lines in Figs. 1 and 2. Any combination of conditions that lie within the V-shaped regions between the dashed and dotted lines in these figures should give rise to SCIP. Comparisons of these predictions with the data points in Figs. 1 and 2 show generally good agreement, although the available measurements are insufficient to fully test the predictions, particularly with regard to the D_T criterion.

It should be noted that we predict that maritime cumulus clouds as shallow as $\sim$300 m can exhibit the OSCIP (Fig. 2) if they are cold enough, whereas, the corresponding minimum depth for continental cumulus is $\sim$1 km (Fig. 1). Also, the OSCIP in cold-based maritime cumulus should occur at much shallower depths than in cold-based continental cumulus.

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*[1] If the reader is uncomfortable with the assumptions and approximations that are inherent in our computed values of D_T, the dashed curves in Fig. 4 can be considered simply as approximate "best-fit" curves to the measurements.

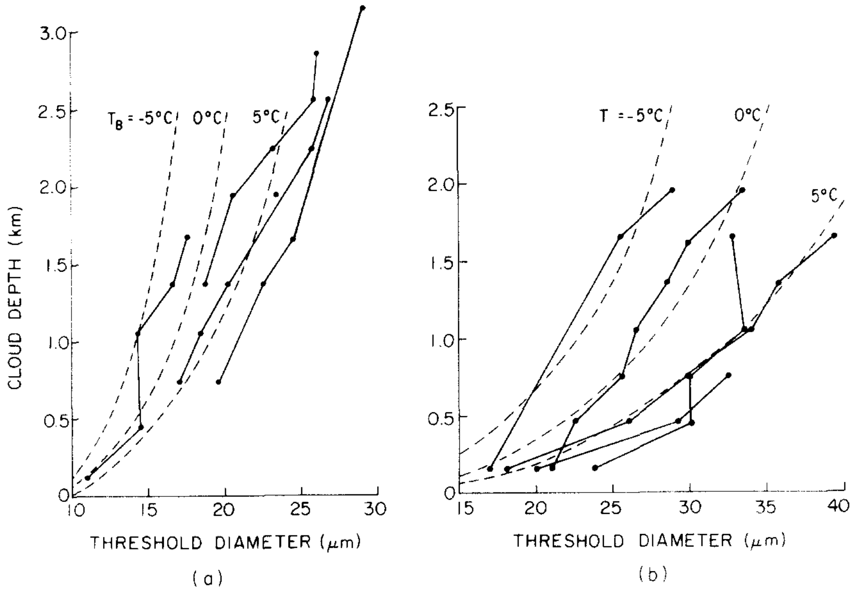

Fig. 4. (a) The solid lines show some of Hobbs and Rangno's (1985) measurements of threshold diameter versus cloud depth for continental cumulus clouds. The dashed lines are computed from eq. 4 for various values of cloud base temperature (T_B). (b) As for (a) but for maritime cumulus clouds.

COMPARISON OF THE HALLETT-MOSSOP AND HOBBS-RANGNO CRITERIA FOR ICE ENHANCEMENT

One of the most widely accepted mechanisms for the occurrence of ice particle concentrations in clouds that are significantly in excess of ice nucleus measurements is through the production of numerous ice splinters during riming. The criteria for splinter production during riming, as revealed by the laboratory experiments of Hallett and Mossop (1974), Mossop and Hallett (1974) and Mossop (1976, 1978b), are: temperatures from $-3°$ to $-8°C$, droplets $\gtrsim 23\,\mu m$ in diameter in concentrations $\gtrsim 1\ cm^{-3}$, and the presence of rimed ice particles with appreciable fallspeeds. Also, increasing numbers of droplets $\lesssim 13\ \mu m$ in diameter increase the efficiency of this process; it becomes most efficient when these small droplets are present in concentrations $\gtrsim 100\ cm^{-3}$ (Hallett et al., 1980). We refer to the above combination of conditions as the Hallett-Mossop (H-M) criteria.

As we have seen in the previous section, the much simpler criteria of H-R appear to be sufficient to describe the present data base on the OSCIP. However, as this data base is expanded by increasing detailed field measurements it should be possible to evaluate whether the H-M or the H-R criteria provide the necessary and sufficient conditions for the OSCIP. In this connection, it is interesting to note that H-M and H-R criteria give rise to some interesting dichotomies. Some examples are given below.

The H-R criteria predict that the OSCIP should occur in tropical maritime cumulus clouds with warm bases and top temperatures of $-7° \pm 3°C$. Hallett et al. (1980), on the other hand, predict that these clouds should exhibit little ice until their tops reach about $-20°C$ (because of a dearth of $< 13\ \mu m$ drops). A recent observation by M. Baker (point 22 in Fig. 2) appears to support the H-R criteria. Also, Kosarev et al. (1978) reported ice in cumulus clouds with top temperature of $-4°$ to $-5°C$ over the eastern tropical Atlantic Ocean.

The H-R criteria predict that cumulus clouds in Israel, with average base temperatures of about 8°C, should exhibit the OSCIP at a cloud depth of 2.1–3.1 km (T_T is $-4°$ to $-10°C$), whereas Gagin (1975) reported the OSCIP at a depth of ~ 3.9 km ($T_T = -17°C$). Gagin's observations are consistent with the H-M criteria since he reports that the clouds he studied lacked both graupel and sufficiently large droplets in the $-3°$ to $-8°C$ temperature range, although droplets $> 30\ \mu m$ diameter existed just above this level. On the other hand, Hobbs and Rangno (1985) documented three clouds that had top temperatures between $-13°$ and $-16°C$ and lacked droplets $\geqslant 23\ \mu m$ diameter in the $-3°$ to $-8°C$ temperature zone, however, these clouds did have droplets $> 20\ \mu m$ in diameter at cloud top and they contained ice particle concentrations of > 1, 15 and $18\ l^{-1}$. Further measurements on cumulus clouds in Israel, and in similar clouds in other parts of the world, will be required to resolve these apparent dichotomies.

The H-R criteria allow the OSCIP to occur in shallow maritime cumulus clouds with base temperatures less than $-8°C$ (see Fig. 2) and one data set (point 25 in Fig. 2) supports this view. However, more data is needed to test this prediction (suitable clouds may be found when clean, arctic air masses flow over cold water, such as the Bering Sea, the northern Gulf of Alaska and the Norwegian Sea). In contrast, the H-M criteria exclude the OSCIP in small clouds with base temperatures less than $-8°C$ but top $\gtrsim -20°C$ or those lacking droplets $\geqslant 24\ \mu m$ in diameter between regions with temperatures from $-3°$ to $-8°C$.

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